Lea Community Primary School

Progression in Calculations

Academic year 2023-2024

Progression in Calculations

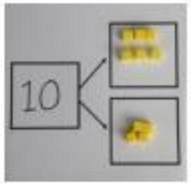
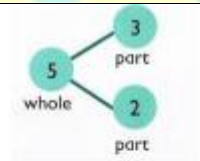
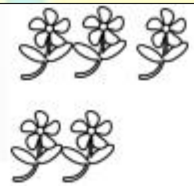
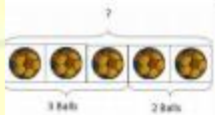
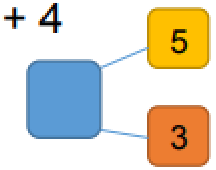
Declarative Knowledge

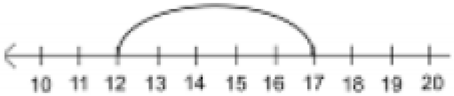
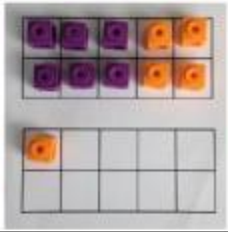
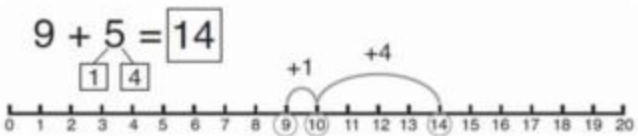
	Additive structures	Multiplicative structures	Fractions
EYFS	**Number bonds to 5**		
Year One	Addition and subtraction within 10.		**Half and one quarter of a quantity**
Year Two	**As above and:** Addition and subtraction across 10.	**Doubles and halves**	**As above and:** **3 quarters of a quantity**
Year Three	**As above and :** Secure and maintain fluency in addition and subtraction within and across 10, through continued practice.	**As above and:** Recall the 10 and 5 multiplication tables, and corresponding division facts. Recall the 2, 4 and 8 multiplication tables, and corresponding division facts	**As above and:** **One third and two thirds of a quantity** **One fifth, two fifths, three fifths, four fifths of a quantity**
Year Four	**Conditional knowledge: Applying the above to problem solving and reasoning.**	**As above and:** Recall the 3, 6 and 9 multiplication tables, and corresponding division facts. Recall the 7 multiplication table, and corresponding division facts Recall the 11 and 12 multiplication tables, and corresponding division facts.	**As above and:** **Equivalent fractions**
Year 5		Secure and maintain fluency in all multiplication tables, and	**As above and:** **Fraction, decimal and percentage**

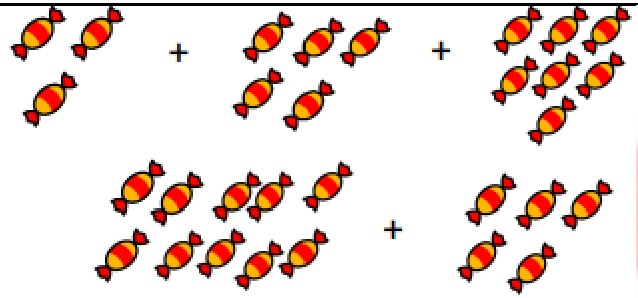
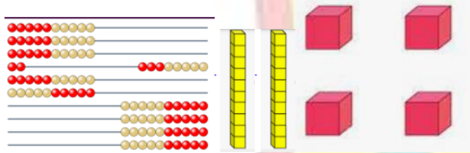
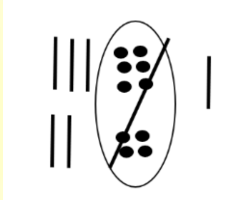
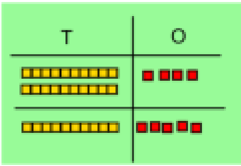
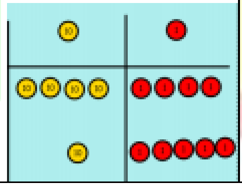
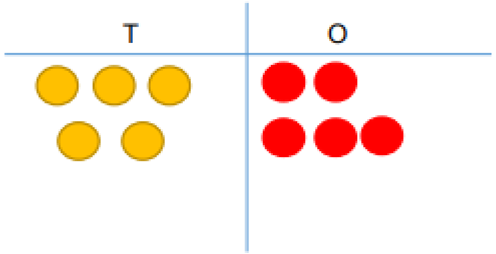
		corresponding division facts, through continued practice.	**correspondence.**
Year 6		**Conditional knowledge: Applying the above to problem solving and reasoning.**	**Conditional knowledge: Applying the above to problem solving and reasoning.**

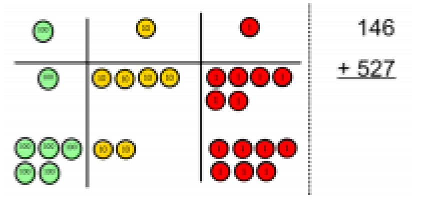
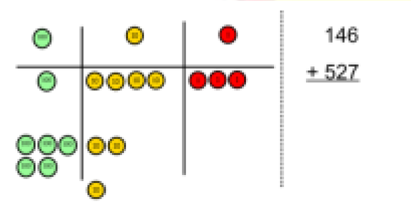
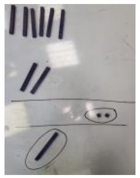
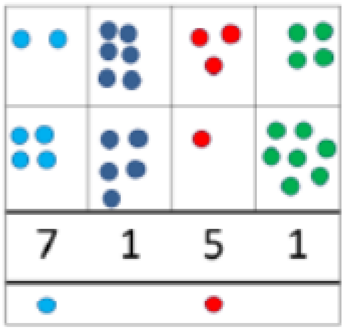
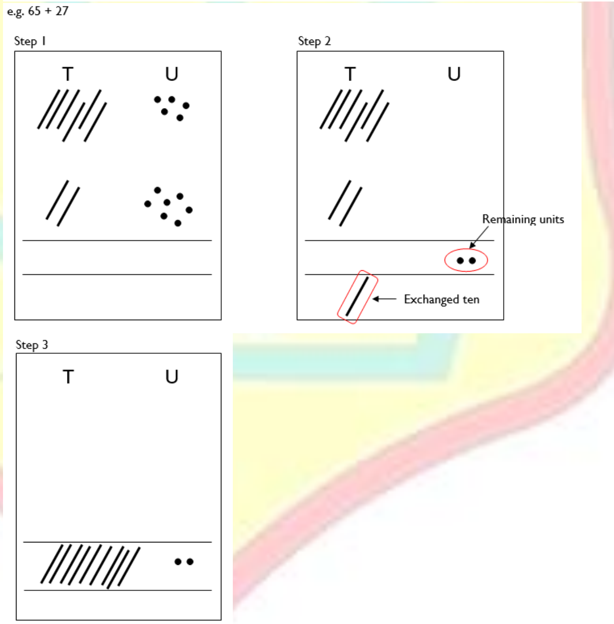
Procedural Knowledge

Addition

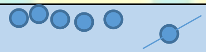
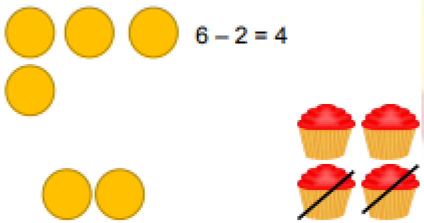
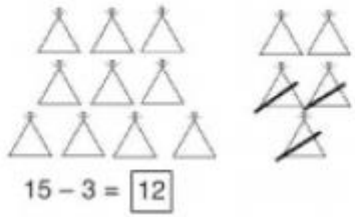
Objective and strategies	Concrete	Pictorial	Abstract
Find one more			One more than 6 is ____ 6 and one more is _______
Combining two parts to make a whole: part whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	$4 + 3 = 7$ $10 = 6 + 4$  Use the part-part whole diagram as shown above to move into the abstract.

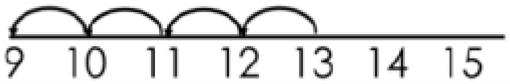
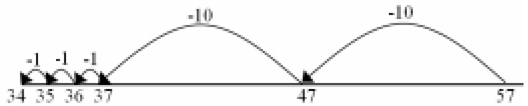
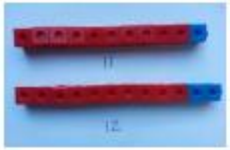
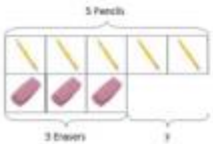
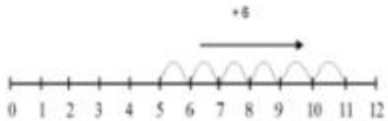
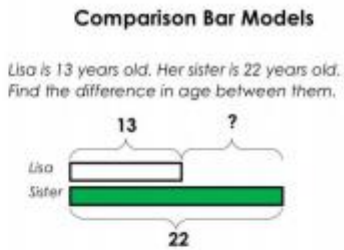
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

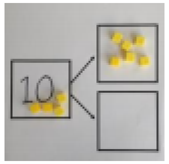
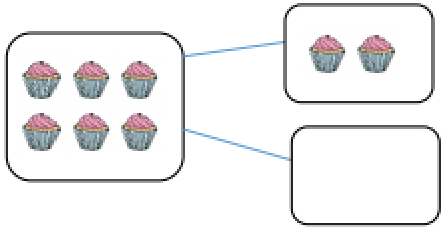
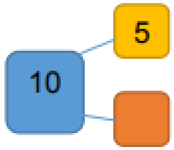
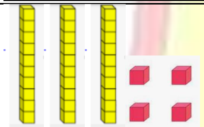
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$\underbrace{4 + 7 + 6}_{10} = \boxed{10} + \boxed{7}$ $= \boxed{17}$ Combine the two numbers that make 10 and then add on the remainder.
Adding two 2 digit numbers, crossing 10			36+24= 25+27= Charlie has 17 marbles. Ahmed has 29. How many do they have altogether.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  	<u>Calculations</u> 21 + 42 = $\begin{array}{r} 21 \\ +\ \underline{42} \end{array}$

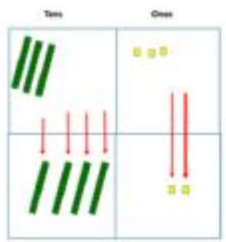
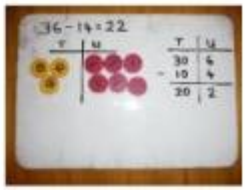
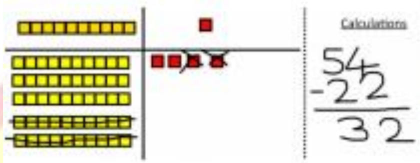
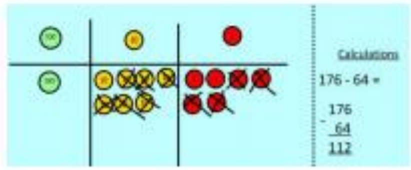
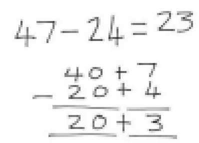
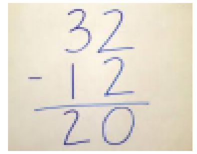
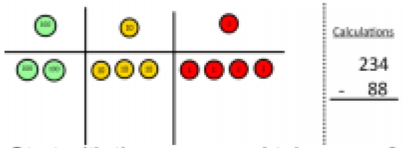
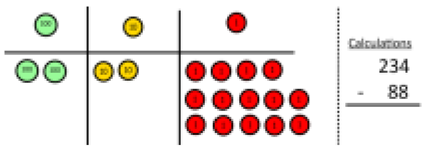
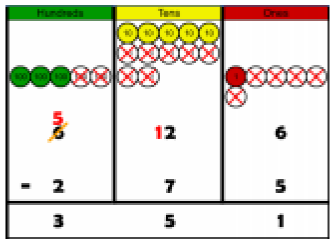
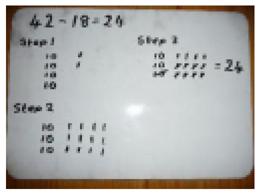
Column method regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.  	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2 Written method Step 1 T U 6 5 + 2 7 Step 2 T U 6 5 + 2 7 2 1 Step 3 T U 6 5 + 2 7 9 2 1

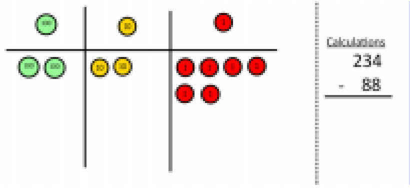
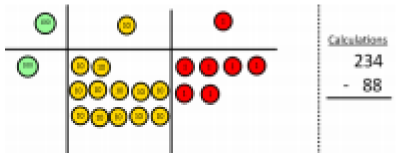
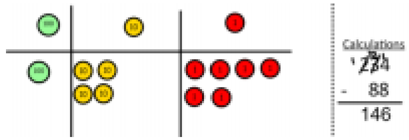
Subtraction

Objective and strategies	Concrete	Pictorial	Abstract
Find one less			One less than 6 is ____ _______is one less than 6.
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  	Cross out drawn objects to show what has been taken away.  	18 -3= 15 8 – 2 = 6

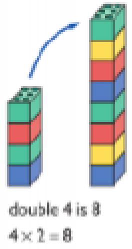
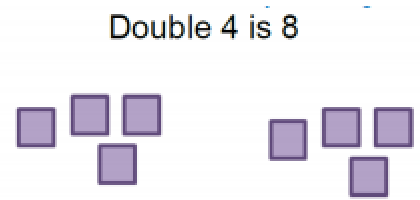
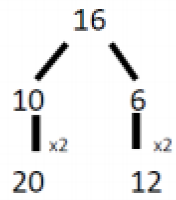
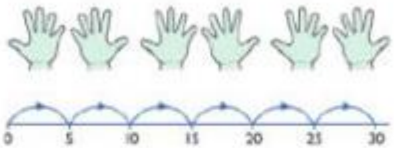
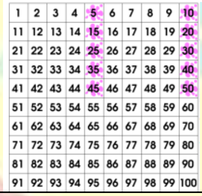
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. 13 – 4 = 9
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. 23 – 15 = 8

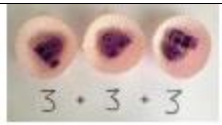
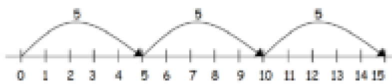
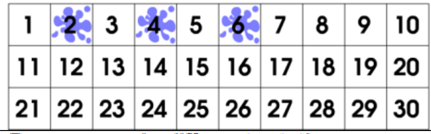
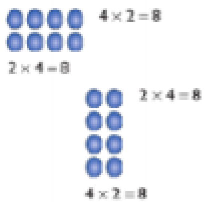
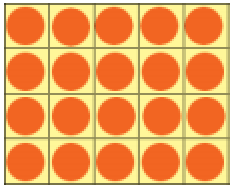
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10 (regrouping)	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Subtract across a 10 (exchanging)			34-5=

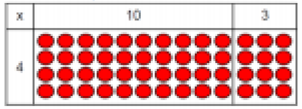
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	 This will lead to a clear written column subtraction. 
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. 	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.

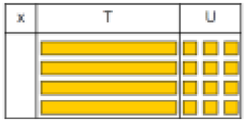
<table>
<tr>
<td></td>
<td>Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

Now I can take away eight tens and complete my subtraction

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.</td>
<td></td>
<td>5 12 1
2 ~~6~~ ~~3~~ . 0
- 2 6 . 5
2 3 6 . 5</td>
</tr>
</table>

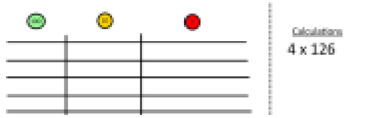
Multiplication

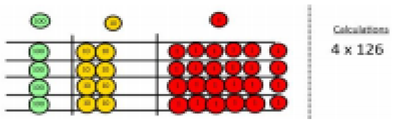
Objective and strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number. 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

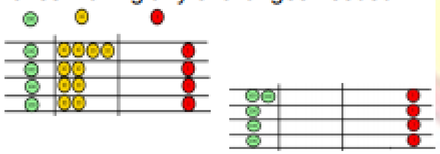
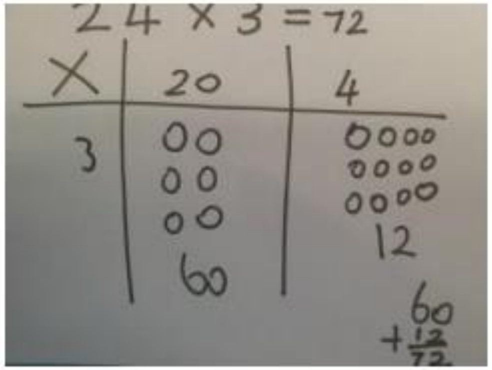
Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15  	Write addition sentences to describe objects and pictures. 
Arrays showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences. e.g. 4 rows of 6 = 24  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

<table>
<tr>
<td>Grid Method</td>
<td>
Show the link with arrays to first introduce the grid method.


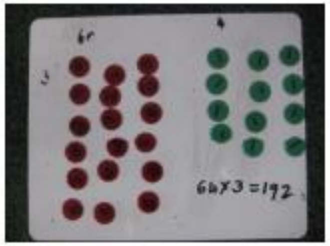
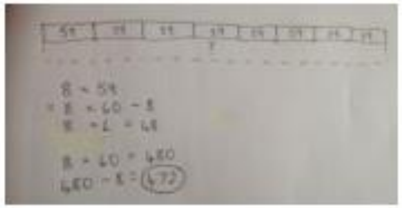
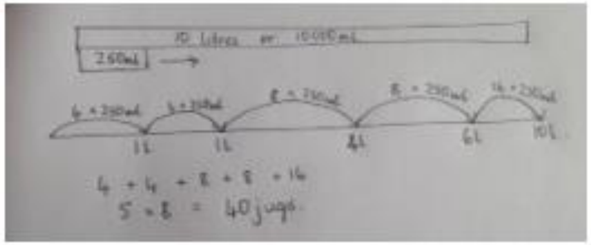
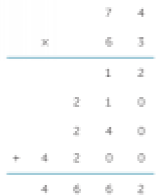
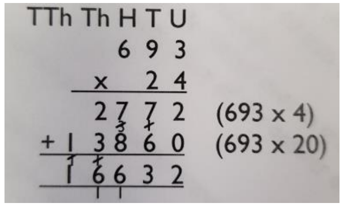
4 rows of 10 4 rows of 3
Move on to using Base 10 to move towards a more compact method.


4 rows of 13
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.


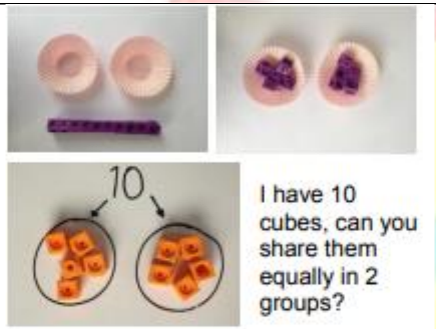
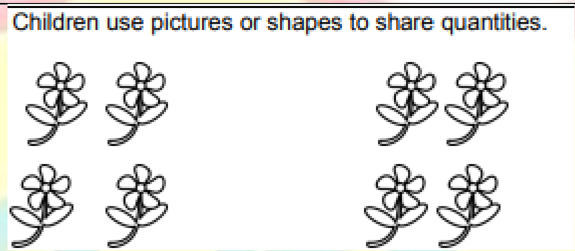
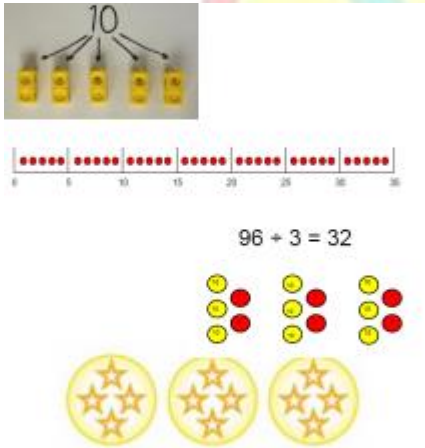
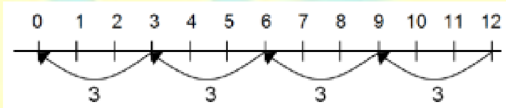
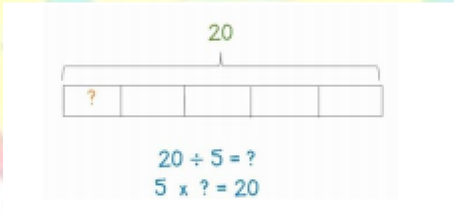
Fill each row with 126.


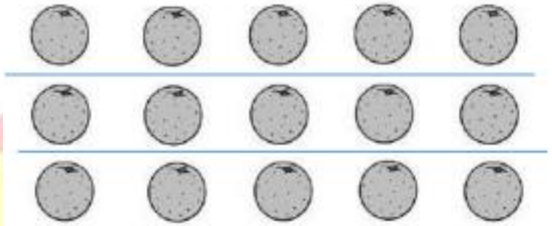
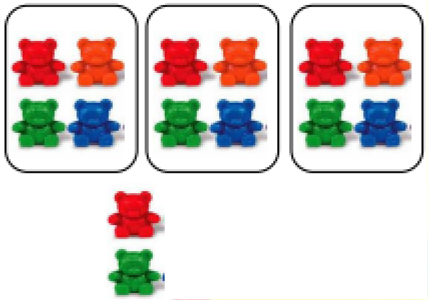
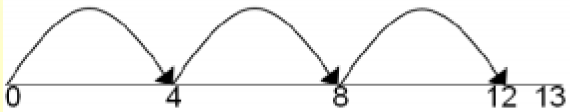
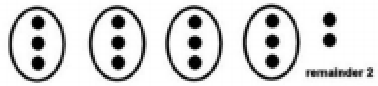
Add up each column, starting with the ones making any exchanges needed.

Then you have your answer.
</td>
<td>
Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.


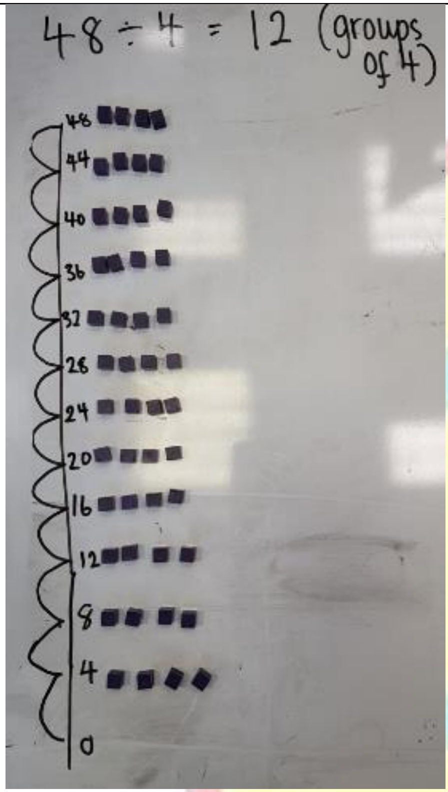
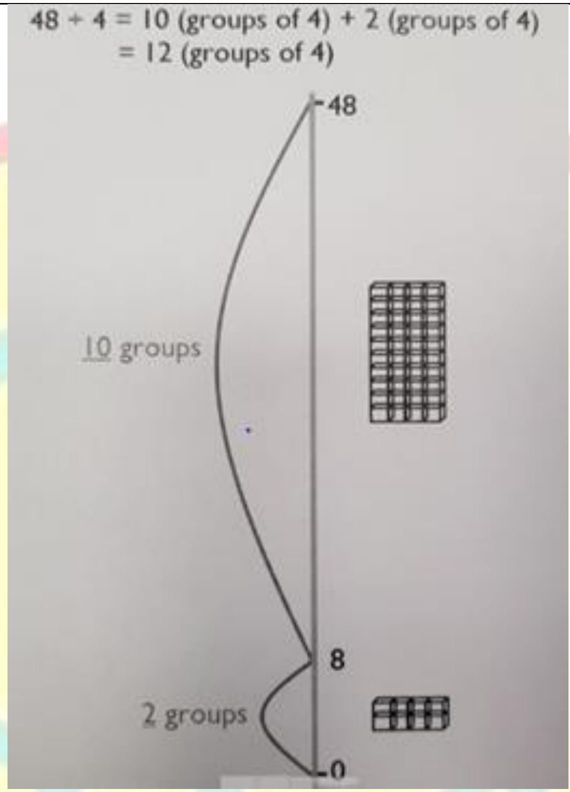
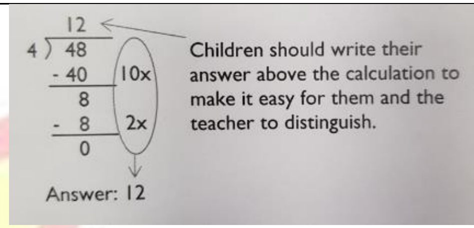
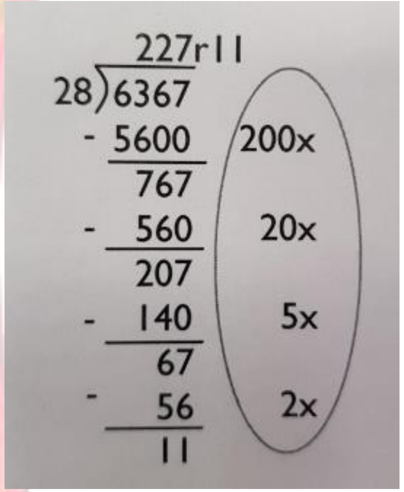
</td>
<td>
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
<table>
<tr><td>x</td><td>30</td><td>5</td></tr>
<tr><td>7</td><td>210</td><td>35</td></tr>
</table>
210 + 35 = 245
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.
<table>
<tr><td></td><td>10</td><td>8</td></tr>
<tr><td>10</td><td>100</td><td>80</td></tr>
<tr><td>3</td><td>30</td><td>24</td></tr>
</table>
<table>
<tr><td>X</td><td>1000</td><td>300</td><td>40</td><td>2</td></tr>
<tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr>
<tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr>
</table>
</td>
</tr>
</table>

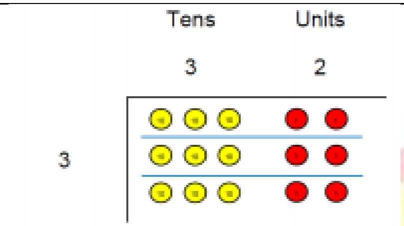
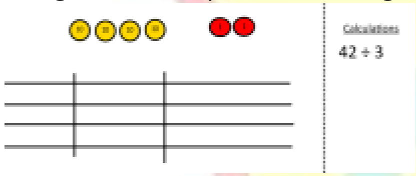
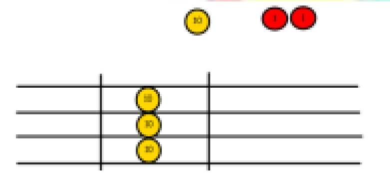
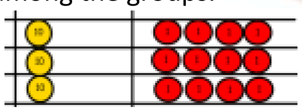
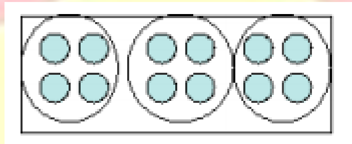
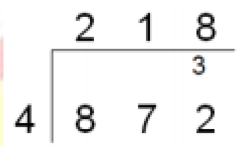
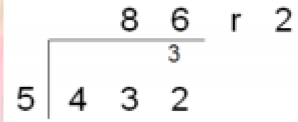
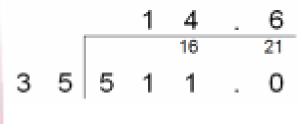
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768  This moves to the more compact method. 

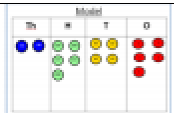
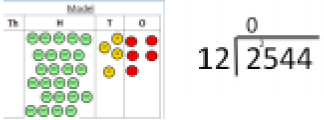
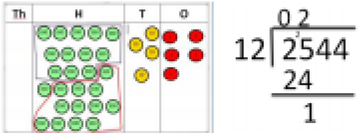
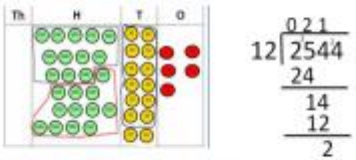
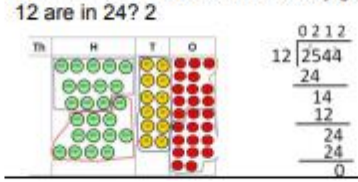
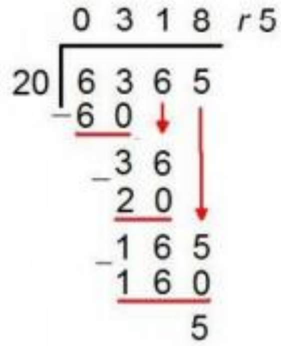
Division

Objective and strategies	Concrete	Pictorial	Abstract	
Sharing objects into equal groups			Share 9 buns between three people.	9÷3=
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?	

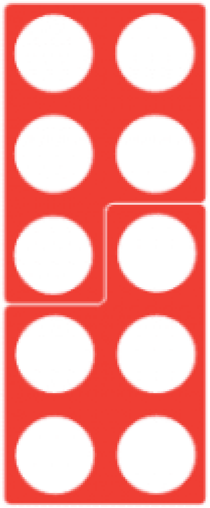
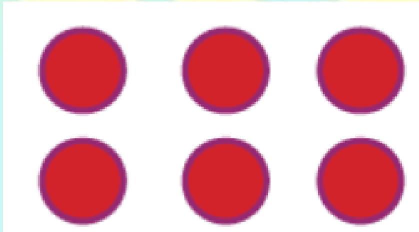
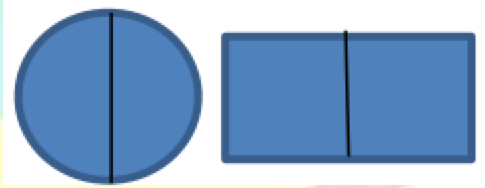
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder

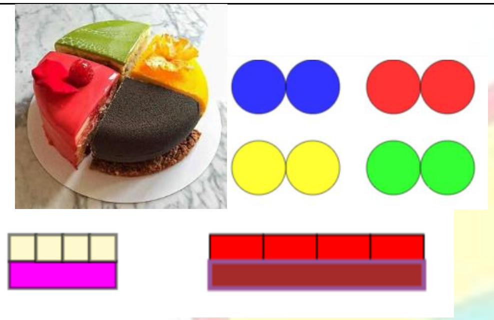
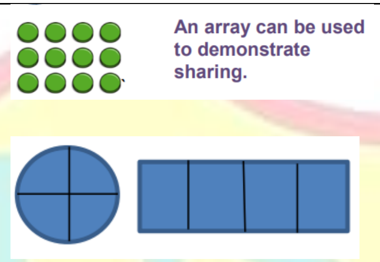
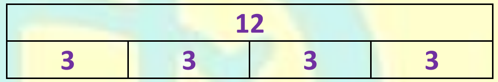
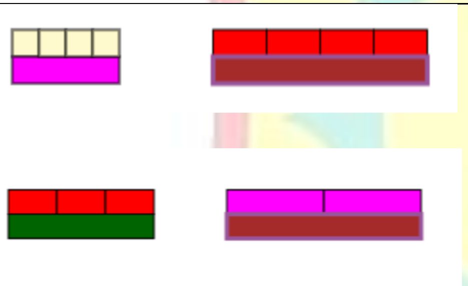
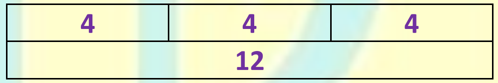
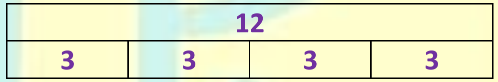
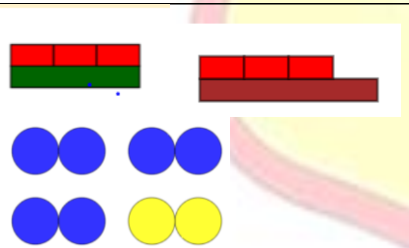
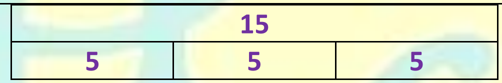
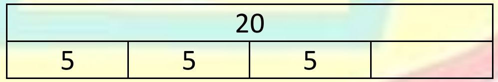
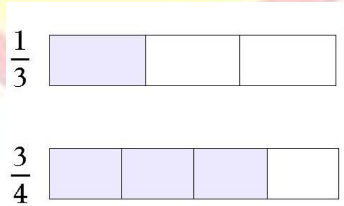
Division by Subtraction (Chunking)	 	 	 Progressing to:  **If children have full conceptual understanding and are proficient with chunking method, they may be taught short (bus stop) and long division.**

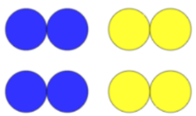
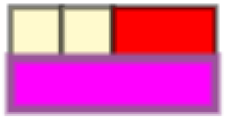
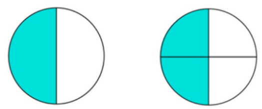
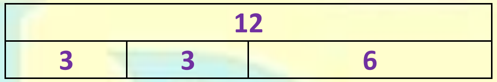
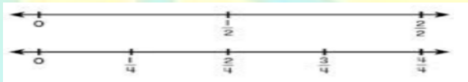
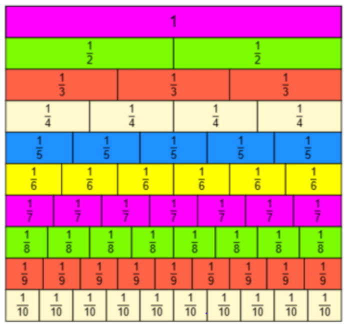
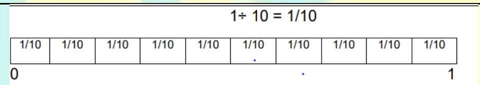
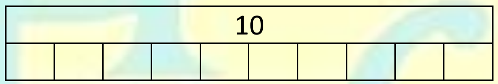
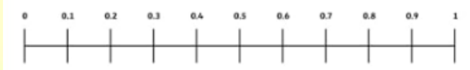
Short division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently	Begin with divisions that divide equally with no remainder  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. 

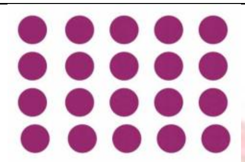
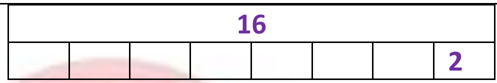
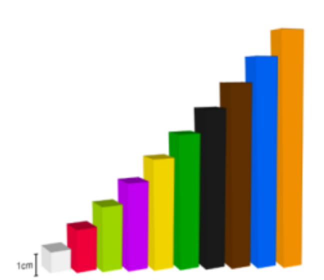
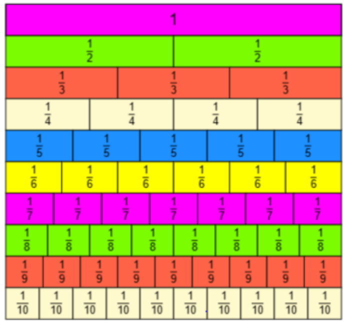
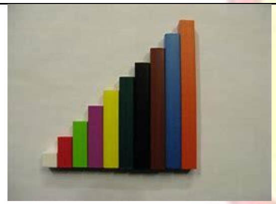
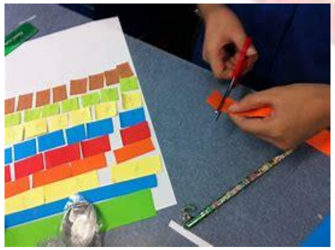
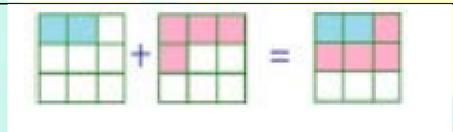
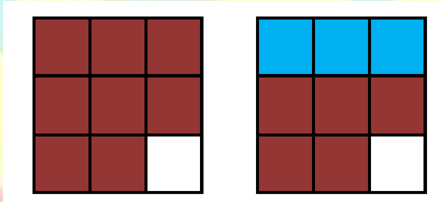
Long division	 2544 ÷ 12 How many groups of 12 thousands do we have? None Exchange 2 thousand for 20 hundreds.  How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one.  Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2  Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2  	Instead of using physical counters, children can draw the counters and circle the groups on a whiteboard or in their books. Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.	

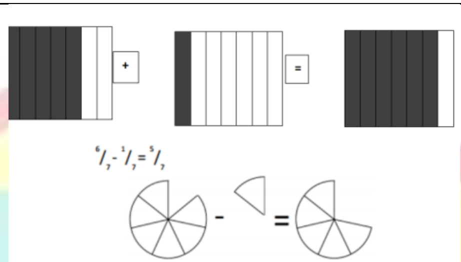
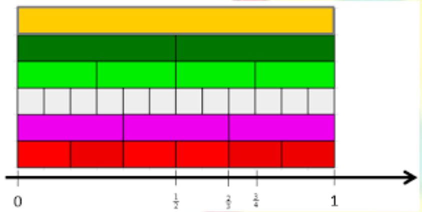
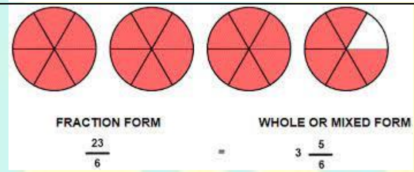
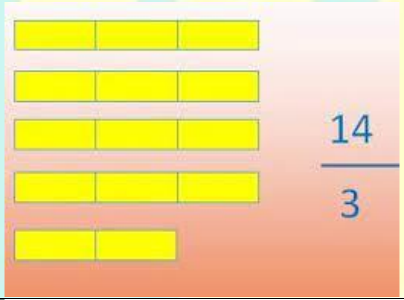
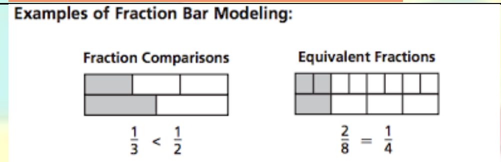
Fractions

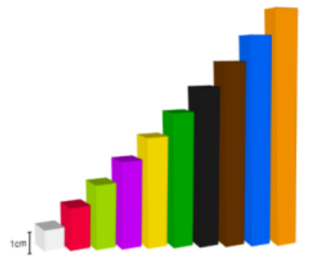
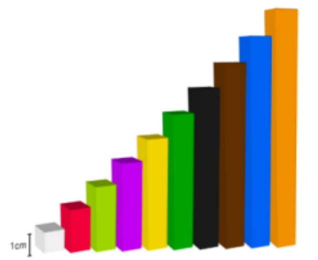
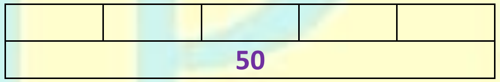
Objective/strategy	Concrete	Pictorial	Abstract
Composition of number: Children will be able to see some numbers can be made of two equal parts.	  	3 | 3 6 	Adults to use fraction vocabulary of halves and quarters accurately
Recognise, find and name a half as one of two equal parts of an object, shape or quantity	  	 6 | 6 12	$\frac{1}{2}$ of 12= **Hamish has 14 bananas. He gives away half. How many does he give away?**

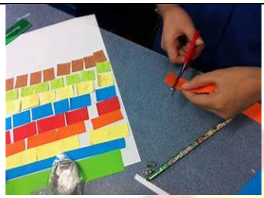
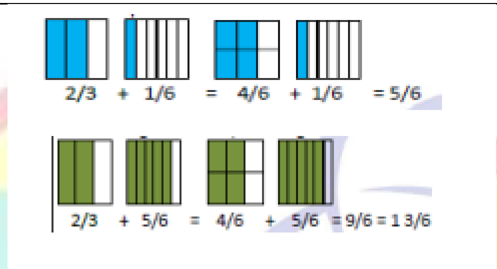
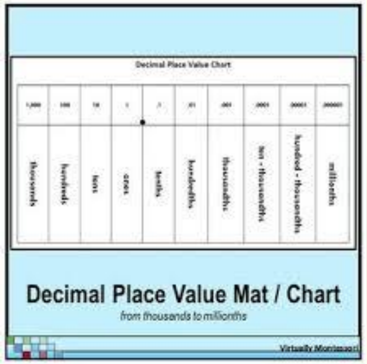
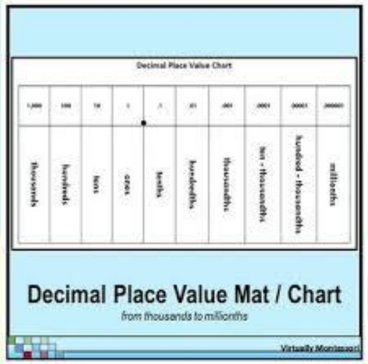
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity Begin to learn sharing and grouping into equal parts.		 	$\frac{1}{4}$ of 20= **Hannah has 20 strawberries. She splits them into quarters. What is a quarter of 20?**
Begin to recognise that the larger the denominator the smaller the fraction (unit fractions or same numerator).		 	$\frac{1}{3}$ of 12=4 $\frac{1}{4}$ of 12=3
Recognise, find, name and write fractions $\frac{1}{3}$, and $\frac{3}{4}$ of a length, shape, set of objects or quantity		  	$\frac{1}{3}$ of 15=5 $\frac{3}{4}$ of 12=9 **Ahmed has 15 cards. He gives away $\frac{1}{3}$. How many does he give away?**

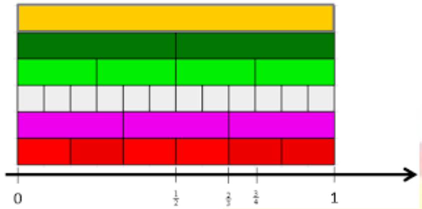
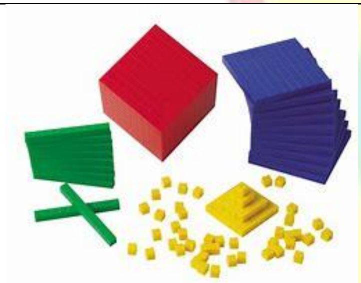
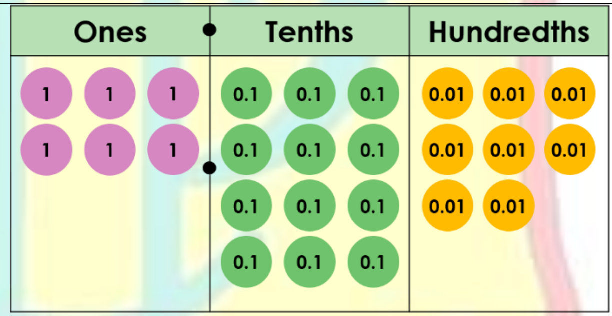
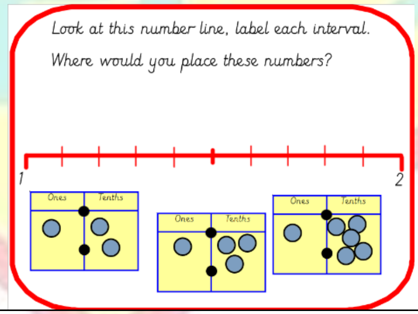
Recognise the equivalence of $\frac{2}{4}$and $\frac{1}{2}$.	 	  	Children need to relate finding a quarter to halving and halving again. They relate this to find fractions of a length e.g. 2/4 of 1m = **$\frac{1}{2}$ of 20=** **$\frac{2}{4}$ of 20=**
Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one digit numbers or quantities by 10	  	  	**$\frac{3}{10}+\frac{1}{10}=$**

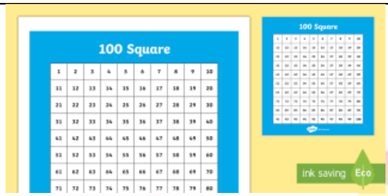
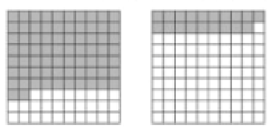
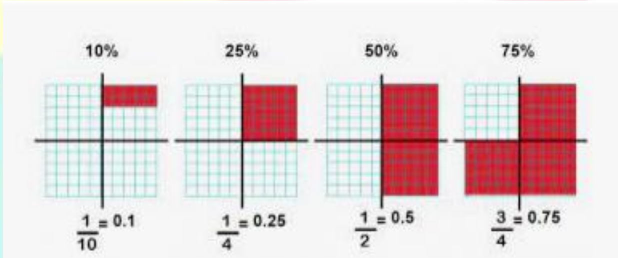
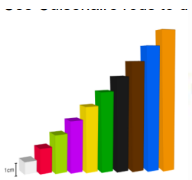
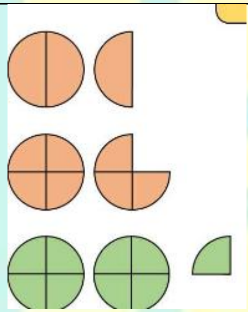
Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators			$\frac{1}{8}$ of 16 $\frac{2}{8}$ of 16
Recognise and show, using diagrams, equivalent fractions with small denominators			$\frac{1}{2} = \frac{}{4}$ $= \frac{}{8}$ $\frac{1}{2} = \frac{}{6}$
Add and subtract fractions with the same denominator within one whole	 	 	$\frac{2}{5}+\frac{1}{5}=$ $\frac{2}{7}+\frac{1}{7}=$ $\frac{4}{5}-\frac{1}{5}=$ $\frac{4}{8}-\frac{1}{8}=$

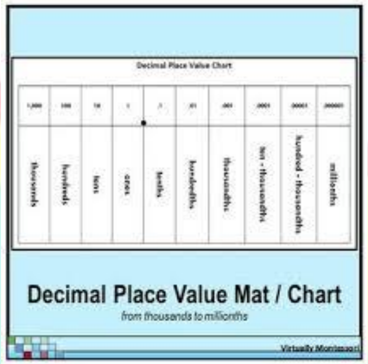
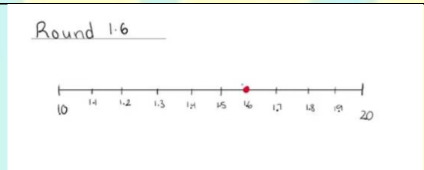
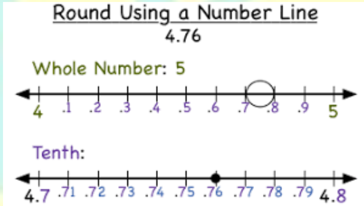
			
Recognise and write mixed number fractions- link to the addition of fractions with the same denominator	 	 	$\frac{4}{5}+\frac{3}{5}=$ $\frac{6}{7}+\frac{5}{7}=$
Compare and order unit fractions	 		$\frac{4}{5} > \frac{1}{3}$ $\frac{2}{8} < \frac{1}{2}$

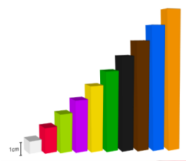
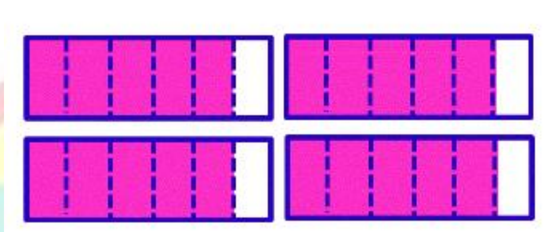
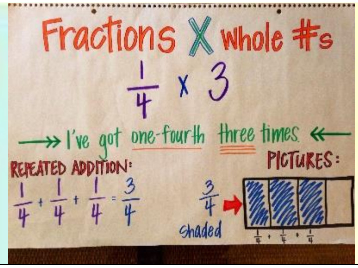
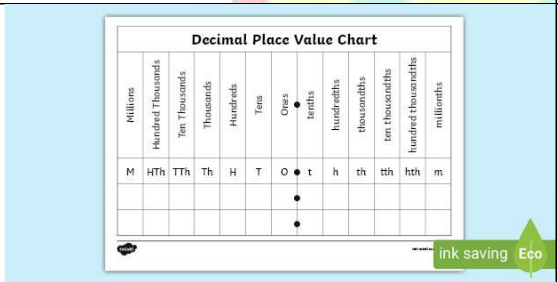
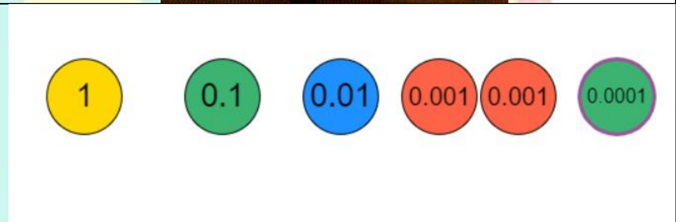
Recognise and show using diagrams, families of common equivalent fractions	 		$\frac{}{5} = \frac{}{15}$ $\frac{2}{3} = \frac{}{6}$ $\frac{4}{8} = \frac{}{4}$
Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number		recognise and use thousandths and relate them to tenths,  hundredths and decimal equivalents	**I have 50 strawberries. I eat $\frac{3}{5}$. How many do I eat? How many do I have left?**

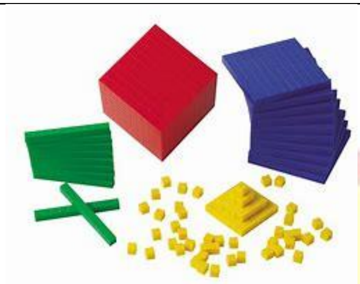
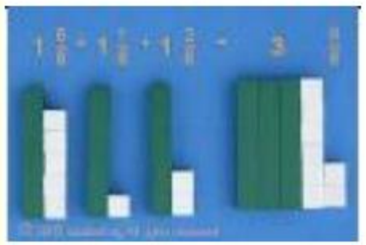
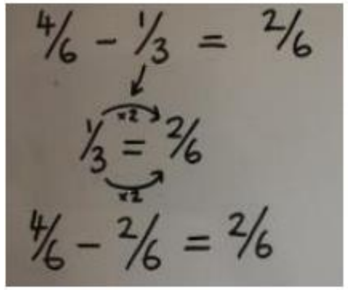
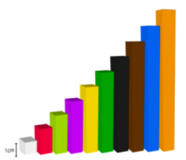
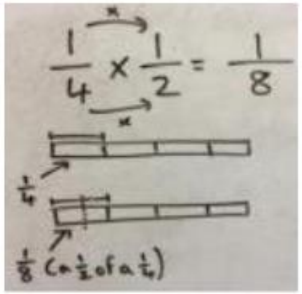
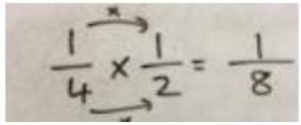
Add and subtract fractions with the same denominator			
Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredth			**45÷10** **45÷100** **67÷10** **67÷100**
Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	 		$\frac{1}{10} = 0.1$ $\frac{4}{10} = 0.4$ $\frac{1}{100} = 0.01$

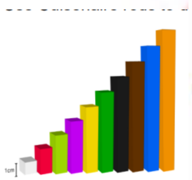
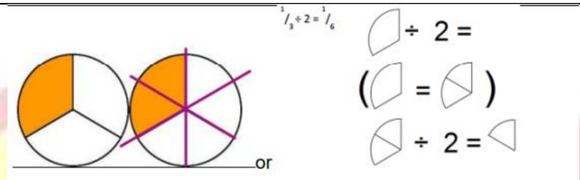
Compare and order fractions whose denominators are all multiples of the same number	 		$\frac{4}{8} < \frac{3}{4}$ $\frac{3}{4} > \frac{1}{2}$
Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents Read and write decimal numbers as fractions	 	 	$0.651 = \frac{651}{1000}$ $1.95 = 1\frac{95}{100}$

Recognise the per cent symbol (%) and understand that per cent relates to “number of parts per hundred”, and write percentages as a fraction with denominator 100 as a decimal fraction	 	 	**0.65=65%** **0.1=10%** **20% of 70=**
Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number	 	 	$1\frac{2}{3} = \frac{5}{3}$ $1\frac{3}{4} = \frac{7}{4}$

Read, write, order and compare numbers with up to three decimal places	 		**0.1 > 0.08 > 0.01** **1 > 0.9 > 0.3 > 0.09**
Round decimals with two decimal places to the nearest whole number and to one decimal place	 	 	**0.67 to the nearest tenth= 0.7** **0.67 to the nearest whole= 1**

Multiply proper fractions and mixed numbers by whole numbers	 	 	$\frac{5}{6}$x4= $\frac{20}{6}$= $3\frac{2}{6}$=
Identify the value of each digit in numbers given to three decimal places			How many thousandths are in 0.543?

	 		
Use common factors to simplify fractions; use common multiples to express fractions in the same denomination		<table><tr><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>	
Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, ¼ x ½ = 1 / 8]	 		

Divide proper fractions by whole numbers	 		$\frac{1}{3} \div 2 = \frac{1}{6}$